The object created by the function `williams` is a matrix. The treatment levels are numbered $1, \ldots, t$ and the entry in the ith row and jth column of the matrix is the treatment the ith sequence group gets in the jth period.

Table 9.6 contains the design from the previous page in a different format. Here sequence groups 1, 2, and 3 are shown as groups 1, and 3 in square I, and sequence groups 4, 5, and 6 are shown as groups 1, 2, and 3 in square II. In this format it can be seen the design is actually composed of two Latin squares since the number of levels of the treatment factor is odd. It can also be seen that the design is balanced for first order carryover effects since each treatment is preceded by every other treatment twice.

Table 9.6 *Williams's Design for Three Treatments*

	Square					
	I			II		
	Period			Period		
Group	1	2	3	1	2	3
1	1	2	3	3	2	1
2	2	3	1	1	3	2
3	3	1	2	2	1	3

As an example of the use of Williams's design for three treatments, consider an experiment conducted by Chipman (2006). The purpose of the experiment was to determine how the surface (grass, cement, or rubberized running track) affected the time to sprint 40 yards. Twelve subjects were recruited for the study, and in order to compare the surface effect within each subject, all subjects ran on all three surfaces. To adjust for the lingering exhaustion effect of each run, the crossover design shown in Table 9.6 was used. Two subjects were randomized to each sequence group in each square. The data resulting from this experiment is the time in seconds for each subject to sprint 40 yards and is shown in Table 9.7. In the following table, treatment level 1 represents cement, treatment level 2 represents the rubberized track, and treatment level 3 represents grass. These surfaces were side by side at the BYU track stadium, which was a convenient location to conduct the experiments.

The R code to get the data (from the data frame `chipman` in the `daewr` package) and fit model (9.3) using the `lm` function is shown below.

```
> library(daewr)
data(chipman)
mod3a <- lm(Time ~ Subject + Period + Treat + Carry, data=chipman)
mod3b <- lm(Time ~ Subject + Period + Carry + Treat, data=chipman)
```

Table 9.7 *Williams's Design and Data for Sprint Time Experiment*

	Square I				Square II			
Group	Subject	Period	Treat	Time	Subject	Period	Treat	Time
1	1	1	1	5.47	7	1	2	5.68
1	2	1	1	6.03	8	1	2	5.90
1	1	2	3	5.00	7	2	3	5.27
1	2	2	3	5.42	8	2	3	5.70
1	1	3	2	5.08	7	3	1	5.23
1	2	3	2	5.38	8	3	1	5.54
2	3	1	2	7.69	9	1	3	5.97
2	4	1	2	6.32	10	1	3	7.87
2	3	2	1	7.03	9	2	1	5.73
2	4	2	1	5.43	10	2	1	6.97
2	3	3	3	7.57	9	3	2	4.97
2	4	3	3	5.77	10	3	2	6.85
3	5	1	3	8.05	11	1	1	6.19
3	6	1	3	7.51	12	1	1	7.39
3	5	2	2	7.12	11	2	2	5.66
3	6	2	2	6.49	12	2	2	6.55
3	5	3	1	7.18	11	3	3	5.57
3	6	3	1	6.35	12	3	3	7.09

Since the level of the carryover effect will always be '0'='none' in period 1, there are missing values (or no responses) in the other levels of the carryover effect in period 1. Therefore the sums of squares for period is not adjusted in the resulting ANOVA table. The treatment sums of squares is adjusted for carryover other terms in the model in `mod3b`, and carryover sums of squares are adjusted for treatments in `mod3a`. The resulting ANOVA table for `mod3b` below shows the treatment factor or running surface caused a significant difference in running times, and the ANOVA table for `mod3a` (not shown) shows that there were significant carryover effects.

```
> anova(mod3b)
Analysis of Variance Table

Response: Time
          Df  Sum Sq Mean Sq F value    Pr(>F)
Subject   11 24.2084 2.20076 85.8462 3.157e-13 ***
Period     2  3.2065 1.60325 62.5388 7.894e-09 ***
Carry      2  0.0217 0.01084  0.4229 0.6615106
Treat      2  0.6392 0.31958 12.4661 0.0004003 ***
Residuals 18  0.4614 0.02564
---
```

```
Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1
```

The R code below produces the estimates of the treatment level means $\hat{\mu}+\hat{\tau}_i$ and standard deviations of data in each treatment level. The results shown below the code indicate that the treatment level 2 (rubberized running track) caused the fastest running times, and that treatment level 3 (grass) produced the slowest running times.

```
> with(chipman, tapply(Time, Treat, mean))
       1        2        3
6.211667 6.140833 6.399167
> sqrt(with(chipman, tapply(Time, Treat, var)))
        1         2         3
0.7646370 0.8398967 1.1248633
```

The R code below produces the estimates of the carryover level means $\hat{\mu}+\hat{\lambda}_l$ in model 9.3.

```
> with(chipman, tapply(Time, Carry, mean))
      0       1       2       3
6.67250 5.97375 6.20250 5.94250
> sqrt(with(chipman, tapply(Time, Carry, var)))
        0         1         2         3
0.9474475 0.9296841 0.8078676 0.8033101
```

Subtracting the grand mean from these estimates, it can be seen that the carryover effects ($\hat{\lambda}_l$ of carryover levels 3 and 1 (grass and cement) are negative which means that the sprinting times will be slightly faster in the period following a sprint on grass or cement. The "0" level of carryover means no prior treatment. The carryover effect ($\hat{\lambda}_l$) for this level is positive indicating the first sprint time is generally longer for each subject (probably to the lack of warm up under race conditions).

9.4.2 Designs with $p > t$ or $t > p$

One problem with Williams's designs is that the direct treatment effects and the carryover effects are not orthogonal. The type III ANOVA table makes a test for significance of treatment effects adjusted for carryover and a test for carryover adjusted for treatments; however, the `lsmeans` package is unable to produce least squares (adjusted) means for treatments and carryover due to the imbalance in these factors. In addition the direct treatment effects